

AMENDMENTS TO THE CLAIMS

Claims 1-18 (Canceled)

19. (New) A method of deriving a set of closed time average turbulent moment equations for anisotropic turbulent flow in an anisotropic fluid comprising:

providing input to a general purpose computer defining a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

instructing the general purpose computer to calculate n^{th} order, wherein n is odd, directional kinetic energy fluxes and structure correlation equations closure relationships using $(n+1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Turbulent Energy

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_1'^2} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1'^2} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1'^2} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1'^2} \right] \\ &+ 2 \left[\overline{u_1'^2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_1' u_2'} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1' u_3'} \frac{\partial \overline{u_1}}{\partial x_3} \right] \\ &+ \frac{1}{\overline{\rho}} \left[\frac{\partial}{\partial x_1} \left[\overline{\rho} \overline{u_1' u_1'^2} \right] + \frac{\partial}{\partial x_2} \left[\overline{\rho} \overline{u_2' u_1'^2} \right] + \frac{\partial}{\partial x_3} \left[\overline{\rho} \overline{u_3' u_1'^2} \right] \right] \\ &= 0 \end{split}$$

Turbulent Shear

004210.P001 -- 8 -- GWH/kew



$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_1' u_2'} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ &+ \overline{u_1' u_2'} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'^2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_2' u_3'} \frac{\partial \overline{u_1}}{\partial x_3} \\ &+ \overline{u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2'} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \\ &+ \frac{1}{\overline{\rho}} \left[\frac{\partial}{\partial x_1} \left[\overline{\rho} \overline{u_2' u_1'^2} \right] + \frac{\partial}{\partial x_2} \left[\overline{\rho} \overline{u_1' u_2'^2} \right] + \frac{\partial}{\partial x_3} \left[\overline{\rho} \overline{u_1' u_2' u_3'} \right] \right] \\ &= 0 \end{split}$$

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_1' u_1'^2} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_1'^2} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_1'^2} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_1'^2} \right] \\ &+ 3 \left[\overline{u_1' u_1'^2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2' u_1'^2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_3' u_1'^2} \frac{\partial \overline{u_1}}{\partial x_3} \right] \\ &- 3 \overline{u_1'^2} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1'^2} \right] + \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3'} \right] \right] \\ &+ 3 \overline{\overline{c_1}^2} \left[\frac{\delta}{\delta x_1} \left[\overline{u_1'^2} \right] + \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3'} \right] \right] \\ &+ 3 \left[\overline{u_1'^2} \frac{\partial}{\partial x_1} \left[\overline{\overline{c_1}^2} \right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_2} \left[\overline{\overline{c_1}^2} \right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3} \left[\overline{\overline{c_1}^2} \right] \right] \\ &+ \frac{\partial}{\partial x_1} \left[\overline{u_1'^2 u_1'^2} \right] + \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2' u_1'^2} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3' u_1'^2} \right] \\ &+ \left[\overline{u_1'^2 u_1'^2} - 3 \overline{u_1'^2} \left[\overline{u_1'^2} \right] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_1} \\ &+ \left[\overline{u_1' u_2' u_1'^2} - 3 \overline{u_1' u_2'} \left[\overline{u_1'^2} \right] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_3} \\ &= 0 \end{split}$$

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_1' u_2'^2} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'^2} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'^2} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'^2} \right] \\ &+ 2 \left[\overline{u_1' u_2'^2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \right] \\ &+ \overline{u_2' u_2'^2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1' u_1'^2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3' u_1'^2} \frac{\partial \overline{u_1}}{\partial x_3} \\ &- 2 \overline{u_1' u_2'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3'} \right] \right] \\ &- \overline{u_2'^2} \left[\frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3'} \right] \right] \\ &+ 2 \left[\overline{c_2^2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \right] \\ &+ 2 \left[\overline{u_2'^2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{c_1^2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \right] \\ &+ 2 \left[\overline{u_1' u_2'} \frac{\partial}{\partial x_2} \left[\overline{c_2^2} \right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3} \left[\overline{c_1^2} \right] \right] \\ &+ 2 \left[\overline{u_1' u_2'} \frac{\partial}{\partial x_2} \left[\overline{c_2^2} \right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_1} \left[\overline{c_2^2} \right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3} \left[\overline{c_1^2} \right] \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} \right] + \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2' u_2'^2} \right] + \frac{\partial}{\partial x_1} \left[\overline{u_1' u_3' u_2'^2} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - 3 \overline{u_1' u_2'} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3' u_2'^2} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1'^2} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3' u_2'^2} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1'^2} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1'^2} \left[\overline{u_1' u_2'} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2' u_2' u$$

$$\begin{split} & \frac{\partial}{\partial t} \left[\overline{u_1' u_2' u_3'} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2' u_3'} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2' u_3'} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_3'} \right] \\ & + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3' u_2'^2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_3'^2} \frac{\partial \overline{u_2}}{\partial x_3} \\ & + \overline{u_3' u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \\ & + \overline{u_2' u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2'^2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \\ & - \overline{u_1' u_2'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_3'} \right] + \frac{\partial}{\partial x_2} \left[\overline{u_2' u_3'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_2' u_3'} \right] \right] \\ & - \overline{u_1' u_3'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3'} \right] \right] \\ & + \overline{u_1' u_3'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_2} \left[\overline{c_1 c_3} \right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3} \left[\overline{c_2 c_3} \right] \\ & + \overline{u_1' u_3'} \frac{\partial}{\partial x_1} \left[\overline{c_1 c_2} \right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_2} \left[\overline{c_1 c_2} \right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_3} \left[\overline{c_1 c_2} \right] \\ & + \overline{u_1' u_3'} \frac{\partial}{\partial x_1} \left[\overline{c_1 c_2} \right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_2} \left[\overline{c_1 c_2} \right] + \overline{u_3' u_3'} \frac{\partial}{\partial x_3} \left[\overline{c_1 c_2} \right] \\ & + \overline{c_1^2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_3'} \right] + \overline{c_1 c_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_3'} \right] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ & + \overline{c_1 c_3} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_3'} \right] + \overline{c_2^2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_3'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ & + \overline{c_1 c_2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_3'} \right] + \overline{c_2^2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_3'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ & + \overline{u_1' u_3' u_1'^2} - \overline{u_1' u_3'} \left[\overline{u_1'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_3'} \right] \right] \frac{\partial}{\rho} \frac{\partial}{\partial x_2} \\ & + \left[\overline{u_1' u_3' u_3'^2} - \overline{u_1' u_3'} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_2' u_3'} \right] \right] \frac{\partial}{\rho} \frac{\partial}{\partial x_3} \\ & + \left[\overline{u_1' u_3' u_3'^2} - \overline{u_1' u_3'} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_3'} \left[\overline{u_2' u_3'} \right] \right] \frac{\partial}{\rho} \frac{\partial}{\partial x_3} \end{aligned}$$

20. (New) A computer readable storage medium containing a set of instructions for a general purpose computer, the set of instructions defining a method of deriving a set of closed time average turbulent moment equations for anisotropic turbulent flow in an anisotropic fluid comprising:

defining a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

calculating n^{th} order, wherein n is odd, directional kinetic energy fluxes and structure correlation equations closure relationships using $(n + 1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Turbulent Energy

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_{1}^{\prime 2}} \right] + \overline{u_{1}} \frac{\partial}{\partial x_{1}} \left[\overline{u_{1}^{\prime 2}} \right] + \overline{u_{2}} \frac{\partial}{\partial x_{2}} \left[\overline{u_{1}^{\prime 2}} \right] + \overline{u_{3}} \frac{\partial}{\partial x_{3}} \left[\overline{u_{1}^{\prime 2}} \right] \\ &+ 2 \left[\overline{u_{1}^{\prime 2}} \frac{\partial \overline{u_{1}}}{\partial x_{1}} + \overline{u_{1}^{\prime} u_{2}^{\prime}} \frac{\partial \overline{u_{1}}}{\partial x_{2}} + \overline{u_{1}^{\prime} u_{3}^{\prime}} \frac{\partial \overline{u_{1}}}{\partial x_{3}} \right] \\ &+ \frac{1}{\overline{\rho}} \left[\frac{\partial}{\partial x_{1}} \left[\overline{\rho} \overline{u_{1}^{\prime} u_{1}^{\prime 2}} \right] + \frac{\partial}{\partial x_{2}} \left[\overline{\rho} \overline{u_{2}^{\prime} u_{1}^{\prime 2}} \right] + \frac{\partial}{\partial x_{3}} \left[\overline{\rho} \overline{u_{3}^{\prime} u_{1}^{\prime 2}} \right] \right] \\ &= 0 \end{split}$$

Turbulent Shear

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_1' u_2'} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ &+ \overline{u_1' u_2'} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'^2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_2' u_3'} \frac{\partial \overline{u_1}}{\partial x_3} \\ &+ \overline{u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2'} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \\ &+ \frac{1}{\overline{\rho}} \left[\frac{\partial}{\partial x_1} \left[\overline{\rho} \overline{u_2' u_1'^2} \right] + \frac{\partial}{\partial x_2} \left[\overline{\rho} \overline{u_1' u_2'^2} \right] + \frac{\partial}{\partial x_3} \left[\overline{\rho} \overline{u_1' u_2' u_3'} \right] \right] \\ &= 0 \end{split}$$

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_{1}' u_{1}'^{2}} \right] + \overline{u_{1}} \frac{\partial}{\partial x_{1}} \left[\overline{u_{1}' u_{1}'^{2}} \right] + \overline{u_{2}} \frac{\partial}{\partial x_{2}} \left[\overline{u_{1}' u_{1}'^{2}} \right] + \overline{u_{3}} \frac{\partial}{\partial x_{3}} \left[\overline{u_{1}' u_{1}'^{2}} \right] \\ &+ 3 \left[\overline{u_{1}' u_{1}'^{2}} \frac{\partial \overline{u_{1}}}{\partial x_{1}} + \overline{u_{2}' u_{1}'^{2}} \frac{\partial \overline{u_{1}}}{\partial x_{2}} + \overline{u_{3}' u_{1}'^{2}} \frac{\partial \overline{u_{1}}}{\partial x_{3}} \right] \\ &- 3 \overline{u_{1}'^{2}} \left[\frac{\partial}{\partial x_{1}} \left[\overline{u_{1}'^{2}} \right] + \frac{\partial}{\partial x_{2}} \left[\overline{u_{1}' u_{2}'} \right] + \frac{\partial}{\partial x_{3}} \left[\overline{u_{1}' u_{3}'} \right] \right] \\ &+ 3 \overline{c_{1}^{2}} \left[\frac{\delta}{\delta x_{1}} \left[\overline{u_{1}'^{2}} \right] + \overline{u_{1}' u_{2}'} \frac{\partial}{\partial x_{2}} \left[\overline{c_{1}^{2}} \right] + \overline{u_{1}' u_{3}'} \frac{\partial}{\partial x_{3}} \left[\overline{c_{1}^{2}} \right] \right] \\ &+ 3 \left[\overline{u_{1}'^{2}} \frac{\partial}{\partial x_{1}} \left[\overline{c_{1}^{2}} \right] + \overline{u_{1}' u_{2}'} \frac{\partial}{\partial x_{2}} \left[\overline{c_{1}^{2}} \right] + \overline{u_{1}' u_{3}'} \frac{\partial}{\partial x_{3}} \left[\overline{c_{1}^{2}} \right] \right] \right] \\ &+ \frac{\partial}{\partial x_{1}} \left[\overline{u_{1}'^{2} u_{1}'^{2}} \right] + \frac{\partial}{\partial x_{2}} \left[\overline{u_{1}' u_{2}' u_{1}'^{2}} \right] + \frac{\partial}{\partial x_{3}} \left[\overline{u_{1}' u_{3}' u_{1}'^{2}} \right] \\ &+ \left[\overline{u_{1}'^{2} u_{1}'^{2}} - 3 \overline{u_{1}' u_{2}'} \left[\overline{u_{1}'^{2}} \right] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_{2}} \\ &+ \left[\overline{u_{1}' u_{3}' u_{1}'^{2}} - 3 \overline{u_{1}' u_{3}'} \left[\overline{u_{1}'^{2}} \right] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_{3}} \\ &= 0 \end{aligned}$$

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_1' u_2'^2} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'^2} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'^2} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'^2} \right] \\ &+ 2 \left[\overline{u_1' u_2'^2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_2' u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \right] \\ &+ \overline{u_2' u_2'^2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1' u_1'^2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3' u_1'^2} \frac{\partial \overline{u_1}}{\partial x_3} \\ &- 2 \overline{u_1' u_2'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3'} \right] \right] \\ &- \overline{u_2'^2} \left[\frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \right] \\ &+ 2 \left[\overline{c_2^2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \right] \\ &+ 2 \left[\overline{u_2'^2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \right] \\ &+ 2 \left[\overline{u_1' u_2'} \frac{\partial}{\partial x_2} \left[\overline{c_2^2} \right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \right] \\ &+ 2 \left[\overline{u_1' u_2'} \frac{\partial}{\partial x_2} \left[\overline{c_2^2} \right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - 3 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \left[\overline{u_1' u_2' u_2'} \right] \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - 3 \overline{u_1' u_2'} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1'^2} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1'^2} \left[\overline{u_2'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1'^2} \left[\overline{u_1' u_2'} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[\overline{u_1' u_2' u_2'^2} - \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_2'} \right] \right] \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_2'} \right] \\ &+ 2 \left[$$

$$\begin{split} &\frac{\partial}{\partial t} \left[\overline{u_1' u_2' u_3'} \right] + \overline{u_1} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2' u_3'} \right] + \overline{u_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2' u_3'} \right] + \overline{u_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2' u_3'} \right] \\ &+ \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3' u_2'^2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_3'^2} \frac{\partial \overline{u_2}}{\partial x_3} \\ &+ \overline{u_2' u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \\ &+ \overline{u_2' u_1'^2} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2'^2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u_2}}{\partial x_3} \\ &- \overline{u_1' u_2'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_3'} \right] + \frac{\partial}{\partial x_2} \left[\overline{u_2' u_3'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_2' u_3'} \right] \right] \\ &- \overline{u_1' u_3'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \frac{\partial}{\partial x_3} \left[\overline{u_1' u_3'} \right] \right] \\ &+ \overline{u_1' u_3'} \left[\frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{u_2' u_2'} \frac{\partial}{\partial x_2} \left[\overline{c_1 c_3} \right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3} \left[\overline{c_2 c_3} \right] \\ &+ \overline{u_1' u_2'} \frac{\partial}{\partial x_1} \left[\overline{c_1 c_3} \right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_2} \left[\overline{c_1 c_3} \right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_3} \left[\overline{c_1 c_3} \right] \\ &+ \overline{u_1' u_3'} \frac{\partial}{\partial x_1} \left[\overline{c_1 c_2} \right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_2} \left[\overline{c_1 c_2} \right] + \overline{u_3'^2} d \frac{\partial}{\partial x_3} \left[\overline{c_1 c_2} \right] \\ &+ \overline{c_1^2} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ &+ \overline{c_1 c_3} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_2'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_2'} \right] + \overline{c_2^2} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ &+ \overline{c_1 c_3} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_3'} \right] + \overline{c_2^2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_3'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ &+ \overline{c_1 c_3} \frac{\partial}{\partial x_1} \left[\overline{u_1' u_3'} \right] + \overline{c_2^2} \frac{\partial}{\partial x_2} \left[\overline{u_1' u_3'} \right] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} \left[\overline{u_1' u_2'} \right] \\ &+ \overline{u_1' u_3' u_3'^2} - \overline{u_1' u_3'} \left[\overline{u_1'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_1' u_3'} \right] \right] \frac{\partial}{\partial \rho} \frac{\partial\rho}{\partial x_3} \\ &+ \left[\overline{u_1' u_2' u_3'^2} - \overline{u_1' u_3'} \left[\overline{u_1'^2} \right] - 2 \overline{u_1' u_2'} \left[\overline{u_2' u_3'} \right] \right] \frac{\partial}{\partial \rho} \frac{\partial\rho}{\partial x_3} \end{aligned}$$

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